Project 1: A Combinatorial Mélange

This is a brief for the first team project in this class. The goal of this project is to gain practical experience enumerating families of combinatorial objects and exploring the connections between them.

Description

In this project, you will explore a family of seemingly unrelated combinatorial objects. Your task is to identify and explore how these objects are related. This will involve ideas from Lessons 1 through 5. For this project, you are not allowed to consult any resources other than the instructor and the assigned readings.

1. To begin, find a formula for the number of each of the following objects for a given natural number n. The elements for $n = 3$ are listed for each set. Make sure to justify each of your answers.

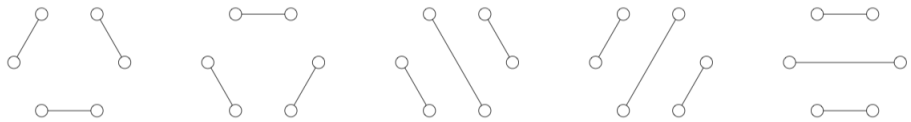
   - Strings of n 0's and n 1's such that no initial segments have more 1's than 0's.

     $$000111 \quad 010011 \quad 010101 \quad 001101 \quad 001011$$

   - The ways of multiplying an ordered list of $n + 1$ numbers together.

     $$(((ab)c)d) \quad ((ab)(cd)) \quad ((a(bc))d) \quad (a((bc)d)) \quad (a(b(cd)))$$

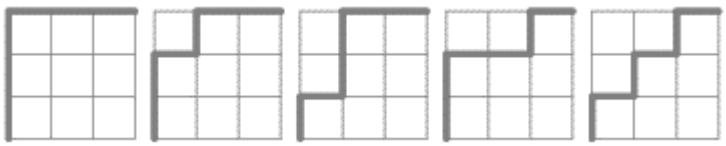
   - The ways in which $2n$ people seated around a circular table can all simultaneously shake hands with another person at the table so that none of their arms are crossing.

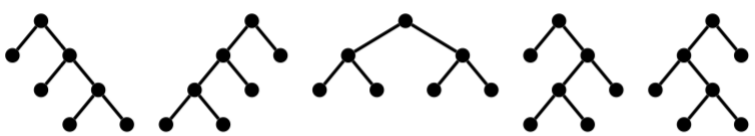
   - The ways of cutting a convex polygon with $n + 2$ sides into triangles without adding new vertices.

   - Paths along the edges of an $n \times n$ grid of square cells, which do not pass below the diagonal.

   - Full rooted binary trees with n internal nodes. (A full rooted binary tree is an arrangement of points, called *nodes*, and line segments, called *edges*, connecting them where there is a special node, called the *root*, and as you descend from the root, there are either two edges going down or zero. *Internal nodes* are the ones that connect to two nodes below.)

- The ways in which the numbers $1, 2, \ldots, 2n$ can be arranged in a $2 \times n$ rectangular grid so that every row and column is increasing.

1	2	3
4	5	6

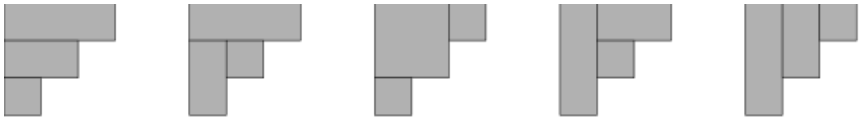
1	2	4
3	5	6

1	2	5
3	4	6

1	3	4
2	5	6

1	3	5
2	4	6

- The ways to tile a stairstep shape of height n with n non-overlapping rectangles.



2. How are the formulas you found in Part 1 related to one another?
3. What properties, if any, do the formulas you found in Part 1 have in common?

⋆. Find more examples of combinatorial objects that belong to this family or construct an analogous family of combinatorial objects of your own. Don't forget to justify your work.

Deliverables

Here is the list of items you will be required to submit as part of this project.

Team Mission Statement

The first deliverable for this project is a *team mission statement* describing how your team will work together, resolve disagreements, and evaluate each other's contributions. The statement should include:

- A name for your team
- Preferred contact information for each member of the team
- A tentative meeting schedule
- A few paragraphs describing how you plan to approach the project, work together, and resolve any disagreements
- A list of guidelines for how your team will interact during team meetings
- A list of criteria for how you will evaluate each other's contributions to the project

Here is a short list of sample guidelines you may wish to include in your mission statement:

- *We recognize that this class is an elective. Everyone who is here wants to be here.*
- *We agree to do our best to make this a brave space to speak openly and share ideas for approaching this project and solving problems.*
- *We agree to listen respectfully to one another in the spirit of learning and to suspend judgement, of ourselves and others.*
- *We will not demean, devalue, nor put down other people for their various experiences, interpretations, and ideas. This includes avoiding the use of micro-aggressions.*

Here is a short list of sample criteria you may wish to include in your peer evaluations:

- *The team member contributed (less/more than) a fair share of ideas to the project.*
- *The team member contributed (less/more than) a fair share of organization to the project.*
- *The team member contributed (less/more than) a fair share of writing to the project.*
- *The team member participated (less/more effectively) during the problem-solving sessions.*

Bring your mission statement signed by every member of your team to class on **23 January 2020**.

Activity Log

To develop good organisation and documentation habits, you should keep a log of all your team activities including the time and duration of each team meeting, who was present, and a brief summary of what was discussed. You should describe the overall process your team used to complete the project and highlight any significant examples, questions, and productive failures (mistakes that led to a deeper understanding) you encountered along the way. You are welcome to include relevant photos of whiteboard work or scans of notebooks, but please be mindful of the file size of the document.

The activity log must be submitted with the final report on **9 February 2020** at 18:00.

Report

This is the main assessment item for the project. You must organise your solutions to Parts 1 through 3 of the Project Description and compose a compendious report on your findings. The report should be typeset and include a title page with an original title, your team name, and the names of your team members; a brief introduction outlining the contents of the report; the main body; and a bibliography (which, in this case, will only include the course textbook since no other references are allowed). The report does not need to include every detail of every idea your team discussed (that's what the activity log is for); instead, it should communicate your key takeaways in a clear and accessible manner.

The report must be submitted with the final report on **9 February 2020** at 18:00.

Reflection & Peer Assessment

At the end of the project, each member of the team must write a reflection on how the project went and prepare an assessment of their teammates' contributions according to the criteria included in the team mission statement. The reflection should describe how you felt throughout the project, what worked well and what did not. It should also identify at least one thing you would have liked to have done better and at least one personal goal for future projects. The peer assessment should include a list of scores for each of your teammates, for each criterion listed in your team mission statement.

For each criterion, you must distribute $2 \times (N - 1)$ *points, where* N *is the number of members in your team, to your teammates. For example, if everyone contributed equally, then everyone should receive* 2 *points. On the other hand, if one team member contributed a lot and another contributed only a little, they should receive* 3 *and* 1 *points, respectively. You may assign any distribution of points you like for each criterion, but you must provide reasonable justification for any distribution that is not uniform.*

The reflection and peer assessment are confidential; they will only be read by the instructor.

The peer assessments and reflections must be submitted on **11 February 2020** at 18:00.

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Rubrics

Team Mission Statement (10%)

The team mission statement will be graded according to the following criteria:

Basic Info The statement should include a team name, contact information and preferred names for each of the team members, and tentative meeting schedule.

- 2 All the requirements are present and clearly stated.
- 1 One or two of the requirements are overlooked or difficult to find.
- 0 Several of the requirements have been overlooked.

Plan The statement should include a plan for how the team will approach the project, work together and resolve disagreements.

- 2 Well-developed, thoughtful, coherent, and detailed.
- 1 Reasonably developed with somewhat vague or impractical components.
- 0 Superficial, vague, or impractical.

Guidelines The statement should include a list of guidelines for how your team will interact during team meetings.

- 2 Well-developed and thoughtful list of guidelines provided.
- 1 Basic list of guidelines provided.
- 0 No guidelines provided.

Evaluation Criteria The statement should include a list of criteria for how the team will conduct peer evaluations at the end of the project.

- 2 Well-developed, practical list of criteria provided.
- 1 Vague or impractical list of criteria provided.
- 0 No criteria provided.

Signatures Every member of the team should agree with and sign the mission statement.

- 1 The mission statement is signed by every team member.
- 0 The mission statement is not signed by every team member.

Timeliness The mission statement should be submitted on time.

- 1 The mission statement was submitted on time.
- 0 The mission statement was submitted late.

Activity Log (30%)

The activity log will be graded according to the following criteria:

Organisation The log should be organised neatly and chronologically.

- 2 Entries easily distinguishable and in chronological order.
- 1 Entries distinguishable with some effort or slightly out of order.
- 0 Entries indistinguishable or significantly out of order.

Completeness The log should include details of all team meetings, when they occurred and what was discussed.

- 2 All team meetings included with detailed summaries.
- 1 A few team meetings or important details overlooked.
- 0 Large segments of the project unaccounted for or undocumented.

Process The log should reflect the process by which your team completed the project.

- 2 Description of process stated clearly and explicitly.
- 1 Evidence of process included implicitly.
- 0 No clear process.

Approaches The log should highlight the different approaches your team considered while completing the project.

- 3 Multiple approaches mentioned and explored.
- 2 Multiple approaches mentioned, but only only one approach explored.
- 1 Only one approach mentioned.
- 0 No approaches mentioned.

Examples The log should highlight important examples your team discussed while completing the project.

- 3 Numerous specific examples mentioned with key ideas synthesised from their exploration clearly acknowledged.
- 2 Several specific examples mentioned with recognised connection to project.
- 1 One or two specific examples mentioned with no attempt to connect them.
- 0 No examples mentioned.

Connections The log should highlight any connections to the course content your team discussed while completing the project.

- 3 Numerous relevant definitions/theorems from the course identified and utilised to complete the project.
- 2 Several relevant definitions/theorems from the course recognised with reasonable attempt to connect them to the project.
- 1 Some relevant definitions/theorems from the course recognised with with no clear attempt to connect them to the project.
- 0 No connections mentioned.

Questions The log should highlight significant questions your team posed while completing the project.

- 3 Several insightful questions raised about the project.
- 2 Several clarification questions raised about statements, definitions, or theorems relevant to the project.
- 1 Recognition of a question indicated, but not stated explicitly.
- 0 No discernible questions included.

Productive Failures The log should highlight any productive failures (mistakes that led to a deeper understanding) your team encountered while completing the project.

- 2 Productive failure mentioned with explanation for why it was productive.
- 1 Productive failure mentioned without explanation for why it was productive.
- 0 No productive failures mentioned.

Report (50%)

The report will be graded according to the following criteria:

Introduction The report should include a compelling and informative introduction that outlines the purpose, content, and structure of the report.

- 3 Compelling, illuminating, and thorough introduction.
- 2 Clear, complete, and accurate introduction.
- 1 Accurate, though somewhat disorganised or incomplete, introduction.
- 0 Introduction overlooked, uninformative, or inaccurate.

Organisation The report should have a clear logical structure that flows naturally from one section to another. It should not be a list of responses to the project prompts.

- 2 Well organised and structured with smooth transitions between sections.
- 1 Discernible structure with some awkward transitions between sections.
- 0 No discernable structure.

Clarity The report should clearly communicate what you did and why you did it.

- 3 No effort required to follow what was done and why.
- 2 Little effort required to follow what was done.
- 1 Some effort required to follow what was done.
- 0 Significant effort required to understand what was done.

Completeness The report should address most of the prompts in the project description.

- 3 The report addresses every prompt in the project description.
- 2 The report addresses most of the prompts in the project description.
- 1 The report only addresses a few of the prompts in the project description.
- 0 The report does not address any of the prompts in the project description.

Depth The report should do more than simply scratch the surface of project topic; it should aim to explore the prompts in depth.

- 3 Insightful exploration and/or demonstration of a solid understanding.
- 2 Substantial exploration and/or demonstration of a good understanding.
- 1 Basic exploration and/or demonstration of a cursory understanding.
- 0 Superficial exploration and/or demonstration of a limited understanding.

Creativity The report should showcase the techniques you used to complete the project.

- 4 New relevant techniques developed for the project.
- 3 Relevant techniques from the course used in a new or original way.
- 2 Relevant techniques from the course used in a conventional way.
- 1 Relevant techniques from the course identified but used deficiently.
- 0 No relevant techniques from the course used.

Correctness The report should be mathematically correct and free of misconceptions.

- 2 No noticeable mistakes or misconceptions.
- 1 A few incorrect/inaccurate statements or misconceptions.
- 0 Many incorrect/inaccurate statements or misconceptions.

Timeliness The report must be submitted on time.

- 2 On time.
- 1 Up to one day late.
- 0 More than one day late.

Peer Assessment & Reflection (10%)

The peer assessment and individual reflection will be graded according to the following criteria:

Participation You will assess each of your team member's participation on the project.

- 1 Assessment completed for all team members.
- 0 Assessment not completed for all team members.

Peer Assessment You will receive a score (out of 3) based on your peers' assessments of you.

Reflection You should include a reflection on how the project went.

- 2 Thoughtful, well-written reflection included.
- 1 Adequate reflection included – ambiguous or superficial in places.
- 0 Inadequate reflection included.

Shortcoming Your reflection should identify something you feel could have gone better.

- 2 Shortcoming identified and thoughtfully explained.
- 1 Shortcoming identified with little or no explanation.
- 0 No shortcoming identified.

Goal Your reflection should include a goal for future projects and a basic plan for how you will achieve it.

- 2 Thoughtful goal(s) included – concrete and practical.
- 1 Reasonable goal(s) included – somewhat tenuous or impractical.
- 0 Impractical or no goal(s) included.